

Academic Scholarship 2009 Preliminary Examination

MATHEMATICS

Time Allowed : 1¹/₂ Hours

No calculating aids may be used.

Solutions are to be written on lined A4 paper.

Please write answers to questions 1 to 5 on one (or more) sheet of paper and then begin a fresh sheet for answering questions 6 to 13. Please staple together sheets of paper used for questions 1 to 5 and similarly for questions 6 to 13. Please then staple both sections together.

Candidates are encouraged to answer questions 1 to 5 to the best of their ability before proceeding to questions 6 to 13. This paper is intended to be challenging. Candidates should not expect to be able to answer all of the questions.

It is vital that candidates show their working and their reasoning.

On questions 6 to 13, a few full solutions will receive better marks than many partial solutions. It is advised, however, that you don't want to spend too long on any one question.

1. Evaluate the following, writing your answers as fractions in their lowest terms:

a.	$\frac{2}{9} + \frac{7}{2}$	
b.	$\frac{3}{23} \times \frac{69}{15}$	
c.	$\frac{4}{7} \div \frac{10}{49}$	
d.	$\frac{1}{2} - \frac{3}{4} \div \frac{5}{6}$	
e.	$\frac{1}{6} \times \frac{2}{7} \times \frac{3}{8} \times \frac{4}{9} \times \frac{5}{10}$	[12]

2. Express the following as fractions in their lowest terms:

a.	32%	
b.	0.86	
c.	$\frac{\sqrt{6^2+8^2}}{4(0.5\times20)}$	
d.	$\frac{(-1)^4}{2^2}$	
e.	$\frac{8-(-17)}{-3-19}$	[12]

3. Substitute p = 3, m = 7, c = -4 into the following expressions and work out their numerical values, leaving the answer as a fraction if desired:

a.
$$pmcc$$

b. $(p+m+c)(c-m-p)$
c. $\frac{(-c)^2}{pm}$
d. $\frac{pm}{mm} \times \frac{m^2}{pc}$
[9]

- 4. Solve the following equations to find all possible values for *x* in each case:
 - a. x 8 = 14

b.
$$39x = 13$$

c.
$$9(x+5) = 81$$

d.
$$4 = \frac{x - (-4)}{-4}$$

e. x + 2x + 3x + 4x + 5x + 6x + 7x + 8x + 9x = 90 [12]

[10]

- 5. Find the next two terms in the following sequences.
 - a. 31,38,45,52, ...
 - b. 4,9,15,22,30, ...
 - c. 1, 3, 6, 10,...
 - d. 0.36, 0.49, 0.64,...
 - e. 0, 2, 4, 6, 0, 6, 6, 2, 8, 0, 8, 8, 6, 4, 0, 4, 4, 8, 2, 0...

- 6. A large cube is constructed from 125 small cubes and sits on the ground. A red man comes along and paints the outside in red paint.
 - a. How many of the small cubes have exactly one face painted red?
 - b. If the man's paint runs out just as he finishes painting, how many small cubes would he have been able to paint?
- 7. I have three hoses and I wish to spraypaint my bathroom. If I use my blue paint hose, I could paint it in 2 hours. If I use my green paint hose, I could do it in 3 hours. If I use my red paint hose, it would take four hours.

If I don't mind very much about what it looks like and use all three hoses at the same time, how long would it take to paint the room?

- 8. 400 people attend a meeting to discuss two ideas: Buses should be painted blue; penguins should be painted red. Surprisingly, 320 people are in favour of buses being blue and 160 are in favour of penguins being red. Even more extraordinarily, 120 are in favour of both ideas! How many people are in favour of neither idea?
- 9. Calculate the value of

$$\frac{1\frac{1}{2}+2\frac{1}{3}}{4\frac{3}{5}}+\frac{1}{6}$$

10.

- a. How many whole numbers have square roots values between 120 and 121?
- b. How many perfect cubes are there between 99 and 999?

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11. Find the missing digits in the following multiplication sum:

		*	*	7	
		3	*	*	
	*	0	*	3	
	*	1	*	*	
*	5	*	*	*	
*	7	*	*	3	

12. To work out $\aleph 8 \square 3 \aleph$ we take the remainder when 8 is divided by 3, i.e. 2.

- When we work out אד 17 ב 7 א the answer is 3, because 7 goes into 17 twice with 3 left over. a. Work out
 - i. א12⊐5×
 - ii. א 60 ב 7 א
 - iii. א 100 ⊐ 13 א
 - iv. ₩85 ב 17 א
 - v. א 30ס א 11 א 30 א א
 - b. Choose a number, N, which makes $\aleph N \supseteq 9 \aleph$ have an answer of 5.
 - c. Find the first number, M, bigger than 100 for which \aleph M \supseteq 11 \aleph has the answer 9.
 - d. Choose a number, P, which makes \aleph P \supseteq 7 \aleph as big as possible.
 - e. We wish to choose T so that $\aleph T \supseteq 3 \aleph + \aleph T \supseteq 5 \aleph$ is as big as possible. What should we choose T to be? Find a second possible value for T.
- 13. A book has suffered some damage. Page 99 exists but then the next page is 444. The book smells of smoke and it is clear that the book has been burning. If it burns one page at a time, takes 10 seconds for one page to burn and it was rescued from burning at exactly 12 noon, what time did it start burning?

END OF EXAMINATION